Exercice 7 :

1. f(x) = ln($x + \sqrt{x^2+2}$).

a) f '(x) = $\frac{1}{\sqrt{x^2+2}}$.

b) I = $\int_0^1 \frac{dx}{\sqrt{x^2+2}} = \left[\ln(x+\sqrt{x^2+2})\right]_0^1$ = ln(1+$\sqrt{3}$) – ln($\sqrt{2}$) = ln $\frac{1+\sqrt{3}}{\sqrt{2}}$.

2. a) J+2I= $\int_0^1 \frac{x^2}{\sqrt{x^2+2}}dx + \int_0^1 \frac{2}{\sqrt{x^2+2}}dx = \int_0^1 \frac{x^2+2}{\sqrt{x^2+2}}dx = \int_0^1 \sqrt{x^2+2}dx$ = K

b) K = $\int_0^1 \sqrt{x^2+2}dx$ On pose u(x) = $\sqrt{x^2+2}$ et v(x) = x

alors u'(x) = $\frac{x}{\sqrt{x^2+2}}$ et v'(x) = 1

d'où K = $\left[x\sqrt{x^2+2}\right]_0^1 - \int_0^1 \frac{x^2}{\sqrt{x^2+2}}dx$ = $\sqrt{3}$ - J , or on a J +2I=K d'où le système :

$\begin{cases} K = \sqrt{3} - J \\ J + 2I = K \end{cases}$ et par suite J = $\sqrt{3} - \frac{\sqrt{2}}{2} - \ln\left(\frac{1+\sqrt{3}}{\sqrt{2}}\right)$ et K= $\frac{\sqrt{2}}{2} + \ln\left(\frac{1+\sqrt{3}}{\sqrt{2}}\right)$

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